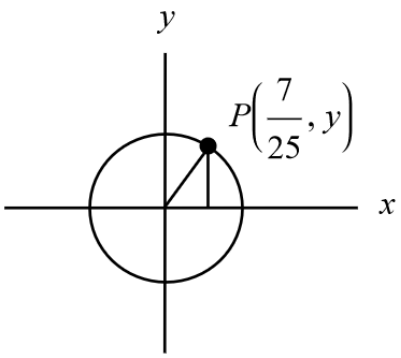
Exercise 135

For the following exercises, P is a point on the unit circle. a. Find the (exact) missing coordinate value of each point and b. find the values of the six trigonometric functions for the angle θ with a terminal side that passes through point P. Rationalize denominators.

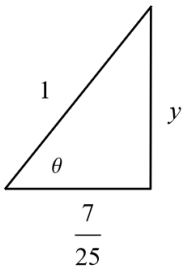
$$P\left(\frac{7}{25}, y\right),\ y > 0$$

Solution

The given point P on the unit circle is shown below. $y > 0$ means that it's in the top half.



Zoom in on the right triangle formed by P. θ is the counterclockwise angle from the positive x-axis.



The hypotenuse has a length of 1 because P is on the unit circle. The sides of a right triangle are related by the Pythagorean theorem, and this allows us to determine y.

$$\left(\frac{7}{25}\right)^2 + y^2 = 1^2$$

$$y^2 = 1^2 - \left(\frac{7}{25}\right)^2$$

$$y^2 = \frac{576}{625}$$

$$y = \frac{24}{25}$$

Therefore, the six trigonometric functions are

$$\sin\theta = \frac{y}{1} = y = \frac{24}{25}$$

$$\cos\theta = \frac{\frac{7}{25}}{1} = \frac{7}{25}$$

$$\tan\theta = \frac{y}{\frac{7}{25}} = \frac{\frac{24}{25}}{\frac{7}{25}} = \frac{24}{7}$$

$$\csc\theta = \frac{1}{y} = \frac{25}{24}$$

$$\sec\theta = \frac{1}{\frac{7}{25}} = \frac{25}{7}$$

$$\cot\theta = \frac{\frac{7}{25}}{y} = \frac{\frac{7}{25}}{\frac{24}{25}} = \frac{7}{24}.$$